Flow Rate in the Discharge of a Two-dimensional Silo.

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Abstract. We present an experimental study of the flow rate in the discharge of a flat bottomed two-dimensional silo. The results of the flow rate dependence on the size of the orifice evidence that the Beverloo expression is not valid for small outlet sizes. This behavior is related with the properties of the flow rate which has been found to fluctuate in a gaussian like form for large orifices. On the contrary, for small orifices extreme events appear at zero flow rates causing a significant slow down of the average flow rate. These events are explained in terms of the existence of arches that block the outlet instantaneously but are unstable to permanently halt the flow.

Keywords: Flow rate, silo, jamming, fluctuations, arch.
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INTRODUCTION

It is well known that the flow rate of grains through the exit of a 3D silo depends on the diameter of the orifice as $D^{5/2}$ [1]. In the same way, in a two dimensional silo, the flow rate is a function of $D^{3/2}$ as indicated in equation 1; where W_s is the average discharge rate through the orifice, D_0 and d_p are the outlet size and the mean diameter of the particles, ρ_b is the apparent density of the granular sample in the bulk, C and k are empirical discharge and shape coefficients, and g is the acceleration of gravity.

$$W_s = C\rho_b\sqrt{g}(D_0 - kd_p)^{3/2} \tag{1}$$

This relationship is known as the Beverloo law and can be obtained by dimensional analysis. In addition, it has been argued that just above the outlet there is a free-fall zone limited by an arch. Above the arch the grains are well packed and their velocities are negligible, whereas below the arch the particles accelerate freely under the influence of the gravity. If the characteristic size of this arch is somehow proportional to the size of the orifice, the velocity of the grains through the outlet of the silo (v_g) can be obtained calculating the velocity of a particle falling without initial velocity from a distance proportional to the size of the outlet: $v_g \propto D^{1/2}$. Provided that in two dimensions $W_s \propto D\,v_g$, the flow rate must be proportional to $D^{3/2}$.

In a recent work important deviations from this behavior have been reported for small orifices in both 2 and 3 dimensional silos [2]. The deviations became apparent when flow rates were measured for a wide region of outlet sizes (spanning over almost two decades in three dimensions and over one decade in two dimensions). These deviations were obtained only for small orifices and did not display any abrupt change in the flow rate values. Then a modification of the Beverloo expression was proposed including an exponential term that allowed an empirical fit of the flow rates for the whole range of outlet sizes:

$$W_b = C'(1 - \frac{1}{2}e^{-b(R-1)})(R-1)^{\frac{3}{2}} \tag{2}$$

where W_b is the flow rate in number of beads per second, C' and b are fitting parameters and R the size of the outlet normalized by the diameter of the particles.

This exponential modification of the Beverloo expression was proved to be valid for experimental results obtained in three and two dimensional silos. Furthermore, numerical results in a two dimensional silo also displayed excellent agreement with the proposed fit. These simulations were used to measure average densities in a region near the outlet which revealed a decrease of the density for small orifices with the same exponential behavior than the deviations from the Beverloo expression. For this reason it was argued that a decrease of the average density near the orifice could be the cause of the flow rate deviations for small orifices.

It is interesting to note that in the aforementioned works the term “flow rate” refers to the mean flow rate during a given lapse of time. Indeed, it is well known that, during the avalanche, the flow rate is not constant as it fluctuates around its mean value [3, 4, 5]. However, there is a lack of knowledge about the nature of the flow rate fluctuations and their dependence on the size of the outlet.

In this work we present experimental results of the flow rate properties for a wide range of outlet sizes. These results evidence the apparition of strong fluctuations in the flow rate for small orifices. These fluctuations –which are not observed for big orifices– will be explained in terms of partial jams of the outlet caused by the development of arches that interrupt the flow tempo-

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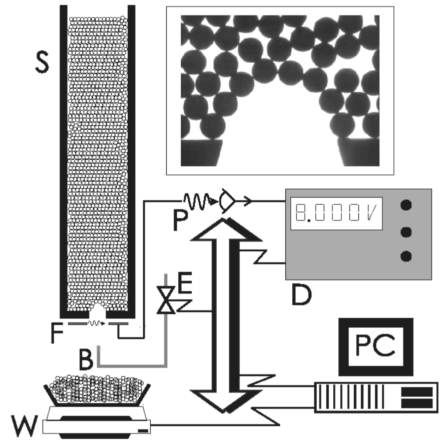

FIGURE 1. Sketch of the experimental setup. S, silo; W, electronic scales; B, blower; E, electrovalve; F, optical fiber; P, photoelectric sensor; D, digitizer; PC, computer. In the inset a photograph of an arch clogging the orifice.

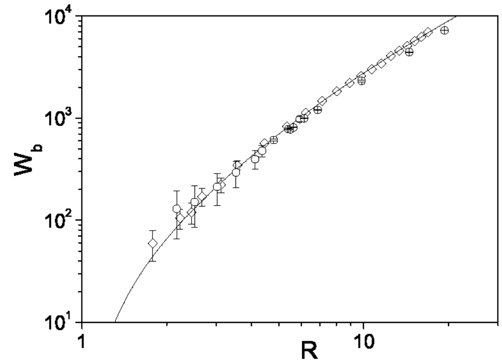

FIGURE 2. Average flow rates obtained experimentally (○) and numericallly (◇) for different outlet sizes (R) in a 2D silo. The error bars are the standard deviations obtained for more than 1000 measurements. The line shows a fit using Eq. 2. The values of the parameters are $C = 108$ and $b = 0.23$.

rally as they are not stable enough to provoke a definite halt of the flow. The formation of such arches has been previously studied both experimentally [6, 7, 8, 9, 10, 11] and theoretically or numerically [12, 13, 14]. Experiments have revealed that in silos of two and three dimensions the formation of an arch that causes a jam (*i. e.* a definitive halt of the flow) strongly depends on the size of the exit orifice. For the two dimensional case, arches that jam the outlet are very common for orifices smaller than about 7 times the diameter of the particle and very rare for orifices above this size.

EXPERIMENTAL SETUP

The experiments were carried out in a flat bottomed two dimensional silo made of two glass panes spaced by two steel stripes and filled with stainless steel spheres of diameter $d = 1.00 \pm 0.01$ *mm*. The distance between the panes was measured to be 1.25 ± 0.05 *mm* which guaranteed a monolayer of particles where the maximum angle of contact respect to the vertical was 7 degrees. The distance between the steel stripes determined the width of the silo which was 200 mm. The size of the exit orifice is adimensionalized by dividing its length by the diameter of the beads: $R = R_0/d$. Further information of the experimental setup can be found in [14].

We define the avalanche size as the number of particles fallen between two jamming events. Then, the average flow rate for each avalanche can be obtained by dividing the number of grains in the avalanche by its duration. The number of grains was obtained by dividing the fallen mass by the weight of a single particle. The scales allowed to measure the avalanche size with a resolution of one particle. The duration of the avalanches was measured with a photodetector. We placed optical fibers at both ends of the slit forming the outlet orifice. Then, a light beam is emitted from one fiber and collected by the other which feeds the photodetector, so a falling bead is detected when it blocks the light beam. The time resolution of this system was better than 1 ms and hence, smaller than the time that it takes for a particle to cross the light beam.

Flow rate measurements were also performed during lapses of time smaller than the duration of the avalanche by recording the region of the outlet with a high-speed camera (Photron Fastcam 1024). Under convenient illumination, each bead reflects a white (or black) spot that can be easily tracked (see inset of Fig. 1). The measurement of the flow rate is carried out by detecting the time at which each particle passes through the outlet by means of a spatiotemporal diagram. This technique, that has been explained in detail in [15], allowed to attain a temporal resolution better than 3×10^{-5} sec. From the times at which the particles pass through the outlet, the flow rate q is calculated as the number of beads that come out from the orifice during a fixed interval of time. The selected time interval was 150×10^{-3} seconds which is two orders of magnitude smaller than the average duration of the avalanche in most cases. In that sense it should be remarked that the flow rate measurements were taken during the discharge process, waiting for 5 seconds after resuming the flow to avoid the transient regime in the flow rate that has been observed at the beginning of the avalanche [3, 4, 5]. The interval of time was also chosen in order to warrant a good precision in the measurement of the flow rate, as during this time, at least several tenths of grains flow trough the outlet in average.

RESULTS

Let us start by presenting in Fig. 2 the results of the average flow rate in number of beads per second (W_b) for different outlet sizes (R). For big outlet sizes the flow rate grows with $R^{3/2}$ as proposed by Beverloo [1]. However, for small orifices clear deviations from this behavior are evident. As explained in [2] the results can be empirically fitted with equation 2 where $C' = 108$ and $b = 0.23$.

The deviations of the experimental results from the Beverloo expression for small outlet sizes suggest that there should be differences in the flow rate properties for small and big orifices. This idea is reinforced when looking at the plots of the avalanche size (s) versus the avalanche duration (ΔT) for different outlet sizes (Fig. 3). It is important to remark that in these graphs, every single point corresponds to a single avalanche and hence the average flow rate obtained for every whole avalanche is $s/\Delta T$. If the outlet is small ($R = 3.5$) the results display a great dispersion which means that, for the same outlet size, the average flow rate takes different values in different avalanches. On the contrary, for greater outlet sizes ($R = 5.4$) the dispersion is vanishingly small and the average flow rate over the whole avalanche is the same for different avalanches. This difference in the average flow rate dispersion for big and small outlet sizes is reflected in the standard deviation which is represented by the error bars shown in Fig. 2. An alternative measurement of the flow rate is through a linear fit of the data displayed in Fig. 3 where the slope gives a mean of the average flow rates obtained for all the avalanches.

Up to now the flow rates presented are average values for each avalanche (or for a long interval of time within an avalanche in the case of big orifices). Consequently, these results do not provide any information about the behavior of the flow rate at short intervals of time. In order to deeply investigate the flow rate properties at different outlet sizes we measured the flow rate within the avalanches (q) at small temporal windows (150×10^{-3} seconds). Examples of typical results obtained within a single avalanche for $R = 4.3$ and $R = 9.5$ are presented in figure 4. In both cases the flow rate fluctuations look similar except for the fact that, for small outlet sizes, there are short time intervals in which the flow goes to zero. These temporary interruptions of the flow are presumably caused by arches that block the exit orifice but are unstable and collapse without any kind of external input of energy. Sometimes these flow interruptions are shorter than the selected temporal window and hence, the flow rate falls to small values but does not cease completely, giving rise to the downward spikes observed in figure 4 for $R = 4.3$. It is important to remark that the spikes always point downwards indicating that extreme events exist for very small values of the flow rate but not for large flow values. In other words, the fluctuations of the flow rate for small orifices are not symmetric.

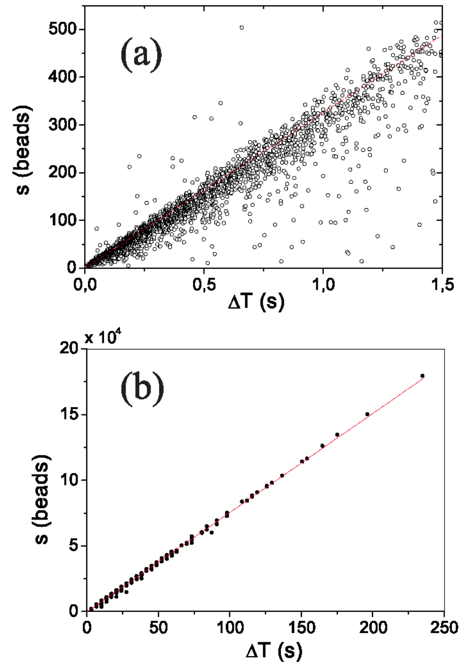

FIGURE 3. Avalanche size (s) in number of particles versus the avalanche duration (ΔT) for two different sizes of the outlet: (a) $R = 3.5$ and (b) $R = 5.4$. The red lines show a linear fit of the data where the slope corresponds to the mean flow rate and takes the values: $W_b = 290 \pm 4$ beads/s for $R = 3.5$ and $W_b = 755 \pm 5$ beads/s for $R = 5.4$

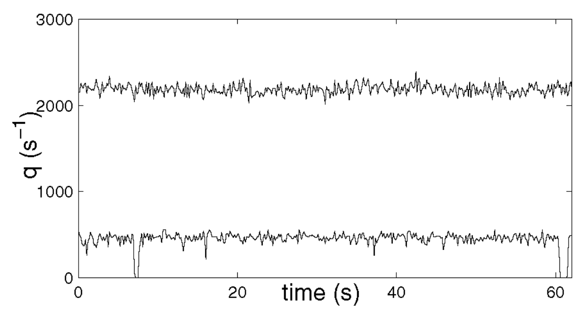

FIGURE 4. Flow rate q (beads per unit time measured at time windows of 150 ms) during a single avalanche for $R = 9.5$ (*top*) and $R = 4.3$ (*bottom*).

The nature of the flow rate fluctuations for different sizes of the exit aperture are clearer observed in the histograms of q displayed in Fig. 5. It is evident that for large orifices, the fluctuations are well fitted by a gaussian. However, for sizes of the orifice below $R = 7.0$,

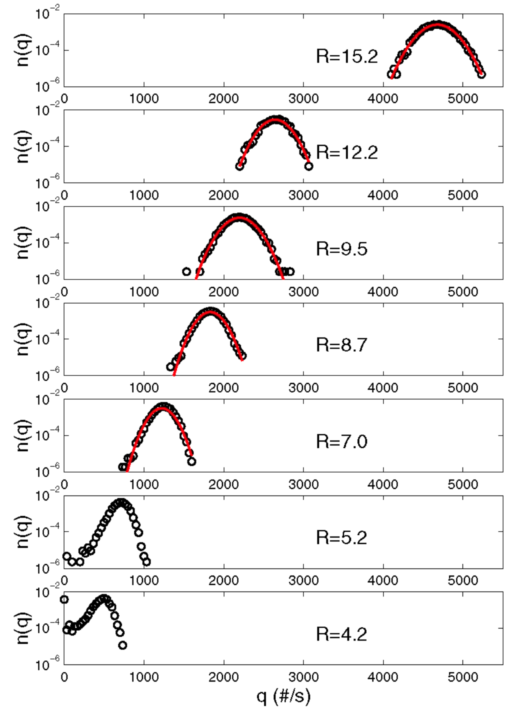

FIGURE 5. Normalized histograms of the flow rate q (in $beads/s$) for different outlet sizes (R). The red line is a gaussian fit, displayed only for the cases where it makes sense.

the fluctuations are not gaussian-like and the histograms seem to be the combination of a gaussian with another set of events at $q = 0$ or q near zero. These events correspond to time intervals during which no beads are falling from the silo due to the formation of unstable arches that block the orifice temporally.

It is interesting to note that the presence of events at $q = 0$ occurs only for small values of R, just in the region where it has been found that jamming is frequent [14] and where the average flow rate deviates from the Beverloo expression. On the contrary, for high values of R –where the probability of observing a jam that completely stops the flow is very low– there are not events at $q = 0$ and the average flow rate is well fitted with the Beverloo expression. The existence of events at $q = 0$ only for small outlet sizes suggests that the temporal interruptions of the flow are also responsible of the great dispersion obtained in the plots of avalanche size versus avalanche duration (Fig 3a). Effectively, the average flow obtained in a single avalanche is strongly conditioned by the number of the events at $q = 0$ that appear within its duration.

CONCLUSIONS

In this work we have studied the flow rate through an orifice at the bottom of a two dimensional silo discharging by gravity. The dependence of the average flow rate on the outlet orifice reveals a clear deviation from the Beverloo expression for small orifices. It has been shown that this deviation is strongly related with the appearance of extreme events in the flow rate measured within the avalanches of small orifices. These extreme events consist on lapses of time during which the flow is temporally interrupted due to the formation of an arch that is unstable and hence unable to halt the flow permanently. Then, it can be stated that for small orifices temporary interruptions of the flow become statistically relevant and provoke a significant reduction of the average flow which deviates from the behavior projected by the Beverloo expression.

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